

Flipped $SU(5)$ From D-branes With Type IIB Fluxes

Ching-Ming Chen^{†1}, V. E. Mayes^{†2}, D. V. Nanopoulos^{†‡§3}

[†]*George P. and Cynthia W. Mitchell Institute for Fundamental Physics,
Texas A&M University, College Station, TX 77843, USA*

[‡]*Astroparticle Physics Group, Houston Advanced Research Center (HARC),
Mitchell Campus, Woodlands, TX 77381, USA*

[§]*Academy of Athens, Division of Natural Sciences,
28 Panepistimiou Avenue, Athens 10679, Greece*

¹`cchen@physics.tamu.edu`, ²`eric@physics.tamu.edu`,
³`dimitri@physics.tamu.edu`

Abstract

We construct flipped $SU(5)$ GUT models as Type IIB flux vacua on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds. Turning on supergravity self-dual NSNS and RR three-form fluxes fixes the toroidal complex structure moduli and the dilaton. We give a specific example of a three-generation flipped $SU(5)$ model with a complete Higgs sector where supersymmetry is softly broken by the supergravity fluxes in the closed string sector. All of the required Yukawa couplings are present if global $U(1)$ factors resulting from a generalized Green-Schwarz mechanism are broken spontaneously or by world-sheet instantons. In addition, the model contains extra chiral and vector-like matter, potentially of mass $\mathcal{O}(M_{string})$ via trilinear superpotential couplings.

1 Introduction

The fundamental goal of string phenomenology is to find a convincing connection between realistic particle physics and string theory. Previously it was thought that only models based upon weakly coupled heterotic string compactifications could achieve this. Indeed, the most realistic model based on string theory may be the heterotic string-derived flipped $SU(5)$ [1] which has been studied in great detail. However, in recent years Type I and Type II compactifications involving D-branes, where chiral fermions can arise from strings stretching between D-branes intersecting at angles (Type IIA picture) [2] and in its T-dual (Type IIB) picture with magnetized D-branes [3], have provided an interesting and exciting approach to this problem.

Many consistent standard-like and grand unified theory (GUT) models were built at an early stage [4, 5, 6, 7, 8] using D-brane constructions. However, these models encountered problems of supersymmetry. Furthermore, these models suffered from instability in the internal space. The quasi-realistic supersymmetric models were constructed first in Type IIA theory on a $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [9, 10, 11] and other orientifolds [12]. Following this, models with standard-like, left-right symmetric (Pati-Salam), Georgi-Glashow ($SU(5)$) and flipped $SU(5)$ gauge groups have been constructed based upon this framework and systematically studied [13, 14, 15, 16].

However, in spite of these successes, a natural mechanism is still needed to stabilize the moduli of the compactification, although in some cases the complex structure parameters (in Type IIA picture) and dilaton fields may be stabilized due to the gaugino condensation in the hidden sector [17]. Turning on RR and NSNS fluxes as background of the compactification gives rise to a non-trivial low energy supergravity potential which freezes some Calabi-Yau moduli [18]. Type IIB configurations with non-trivial RR and NSNS fluxes together with the presence of anti-D3 branes have been studied in [19, 20]. These fluxes impose strong constraints on the RR tadpole cancellation by giving large positive D3 RR charges since their supergravity equation of motion and the Dirac quantization conditions must be satisfied.

In the closed string sector, generic choices of the fluxes do not preserve supersymmetry. This leads to soft supersymmetry breaking terms at a mass scale $M_{soft} \sim \frac{M_{string}^2}{M_{Pl}}$ which implies an intermediate string scale or an inhomogeneous warp factor in the internal space to stabilize the electroweak scale [21, 22, 23]. On the other hand, for the string scale to be close to the Planck scale, supersymmetry in the open string sector must be preserved by fixing the Kähler toroidal moduli [23], which is T-dual to the supersymmetry consistency conditions in Type IIA theory. Recently D-brane constructions corresponding to models with magnetized D-branes where the role of the intersection angles is played by the magnetic fluxes on the D-branes on the Type IIB $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold have been studied [23, 24, 25, 26, 27].

As previously mentioned, there are only a few specific choices of fluxes which are supersymmetric in the closed string sector, which is interesting from a phenomenological point of view. In general, non-supersymmetric fluxes lead to soft supersymmetry breaking terms in the effective action of open string fields. Detailed studies of the soft-breaking mechanism and some trial investigations in the effective low energy scenario were explored in [22, 28]. Combined with an analysis of the Yukawa couplings [29], these studies may

provide a clear picture of the low energy physics in the intersecting D-brane configuration which is worthwhile for future work. On the other hand, if supersymmetry is required to be conserved both in the closed and open string sectors, it has been recently shown that the RR, NSNS and metric fluxes could contribute negative D6-brane charges in Type IIA orientifold with flux compactifications, which makes it easier to satisfy the RR tadpole cancellation conditions [30]. We will presently not consider this, but plan to investigate this possibility in the future.

In this paper we search for consistent flipped $SU(5)$ models on a Type IIB $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with supergravity fluxes turned on. As mentioned before, due to the difficulty to impose supersymmetric fluxes in the closed string sector with consistent RR tadpole conditions, we do not insist that the fluxes be supersymmetric and consider all possible fluxes in constructing flipped $SU(5)$ models. However, supersymmetry in the open string sector is still preserved for a reasonable string scale. By requiring that the gauge bosons coupled to $U(1)_X$ do not acquire a string scale mass via a generalized Green-Schwarz mechanism, which has four constraints in $(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold construction, we find that the models must have at least five stacks of D-branes. In addition, there are K-theory constraints which must be imposed to avoid the anomaly classified by the discrete symmetry \mathbb{Z}_2 . Some K-theory properties are modified by the NSNS fluxes, but this is currently regarded to have no effect on phenomenology [25].

Next, we turn to the question of our motivation in building flipped $SU(5)$ models. Different types of particle models have been discussed using various constructions. The minimal option is to embed just the Standard Model $SU(3) \times SU(2) \times U(1)$ gauge group, but almost every construction contains at least some extra $U(1)$ factors. Conventional GUT models such as $SU(5)$ or $SO(10)$ have been investigated, but none of them has been completely satisfactory. This triggered the motivation to consider the gauge group $SU(5) \times U(1)_X$ [1, 31, 32] as a candidate for a model derived from string. The *raison d'être* of this ‘flipped’ $SU(5)$ is that it requires only **10** and $\overline{\mathbf{10}}$ Higgs representations to break the GUT symmetry, in contrast to other unified models which require large and unwieldy adjoint representations. This point was given further weight when it was realized that models with adjoint Higgs representations cannot be derived from string theory with a $k = 1$ Kac-Moody algebra [33]. There are many attractive features of flipped $SU(5)$. For example, the hierarchy problem between the electroweak Higgs doublets and the color Higgs triplets is solved naturally through a ‘missing partner’ mechanism [1]. Furthermore, this dynamical doublet-triplet splitting does not require or involve any mixing between the Higgs triplets leading to a natural suppression of dimension 5 operators that may mediate rapid proton decay and for this reason it is probably the simplest GUT to survive the experimental limits placed upon proton lifetime [34]. Recent investigation showed that the proton could be even stable by rotating away the gauge dimension 6 contributions [35]. More recently, the cosmic microwave anisotropy $\delta T/T$ has been successfully predicted by flipped $SU(5)$, as it has been determined to be proportional to $(M/M_P)^2$ where M denotes the symmetry breaking scale and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass [36]. Finally, string-derived flipped $SU(5)$ may provide a natural explanation for the production of Ultra-High Energy Cosmic Rays (UHECRs), through the decay of super-heavy particles dubbed ‘cryptons’ [37] that arise in the hidden sector of the model, which are also candidates for cold-dark matter (CDM).

The heterotic string-derived flipped $SU(5)$ model was created within the context of

the free-fermionic formulation, which easily yields string theories in four dimensions. This model belongs to a class of models that correspond to compactification on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold at the maximally symmetric point in the Narain moduli space [38]. Although formulated in the context of weakly coupled heterotic string theory, it is believed that the vacuum may in fact be non-perturbative due to its proximity to special points in the moduli space and may elevate to a consistent vacuum of M-theory. For this reason, it is our hope that in searching for a realistic flipped $SU(5)$ model that we may arrive at or near the same vacuum using D-brane constructions.

We organize this letter in the following way. In section 2 a brief but complete construction of D-branes compactified on $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with Type IIB RR and NSNS supergravity fluxes is provided. In section 3 a short review of basic flipped $SU(5)$ phenomenology is presented. Section 4 contains the discussion of D-brane model building with fluxes, and we provide a few examples including a complete spectrum of a flipped $SU(5)$ model. We present our conclusions in section 5.

2 D-branes with Type IIB Flux on the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

2.1 Magnetized D-branes in Type IIB Theory

We begin with the Type IIB theory on the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where \mathbf{T}^6 is product of three two-tori and the two orbifold group generators θ, ω act on the complex coordinates (z_1, z_2, z_3) as

$$\begin{aligned}\theta &: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)\end{aligned}\tag{1}$$

This construction contains a $D = 4, N = 2$ supergravity multiplet, the dilaton hypermultiplet, h_{11} hypermultiplets, and h_{21} vector multiplets which are all massless. For the orbifold with discrete torsion the Hodge numbers from both twisted and untwisted sectors are $(h_{11}, h_{21}) = (3, 51)$. In order to include the open string sector, orientifold planes are introduced by an orientifold projection ΩR , where Ω is the world-sheet parity and R acts as

$$R : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3)\tag{2}$$

There will then be 64 $O3$ -planes and 4 $O7_i$ -planes, which are transverse to the \mathbf{T}_i^2 . Thus ΩR projects the $N = 2$ spectrum to an $N = 1$ supergravity multiplet, the dilaton chiral multiplet, and 6 untwisted and 48 twisted geometrical chiral multiplets. [23, 26]

We need $D(3 + 2n)$ -branes to fill up the four-dimensional Minkowski space-time and wrapping the $2n$ -cycles on a compact manifold in type IIB theory. The introduction of magnetic fluxes provides more flexibility in constructing models. For one stack of N_a D-branes wrapping m_a^i times on \mathbf{T}_i^2 , n_a^i denotes the units of magnetic fluxes F_a^i turned on each \mathbf{T}_i^2 , thus

$$m_a^i \frac{1}{2\pi} \int_{\mathbf{T}_i^2} F_a^i = n_a^i\tag{3}$$

To write down an explicit description of D-brane topology we introduce the even homology classes $[\mathbf{0}_i]$ and $[\mathbf{T}_i]$ for the point and the two-torus. Then the vectors of RR charges (corresponding to Type IIA homology cycles) of a^{th} stack D-brane and its image are [25]

$$[\Pi_a] = \prod_i^3 (n_a^i [\mathbf{0}_i] + m_a^i [\mathbf{T}_i]), \quad [\Pi'_a] = \prod_i^3 (n_a^i [\mathbf{0}_i] - m_a^i [\mathbf{T}_i]) \quad (4)$$

The O3- and O7_i-planes of $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ resulting from the orientifold action ΩR , $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ can be written as

$$\begin{aligned} \Omega R : \quad & [\Pi_{O3}] = [\mathbf{0}_1][\mathbf{0}_2][\mathbf{0}_3] \\ \Omega R\omega : \quad & [\Pi_{O7_1}] = -[\mathbf{0}_1][\mathbf{T}_2^2][\mathbf{T}_3^2] \\ \Omega R\theta\omega : \quad & [\Pi_{O7_2}] = -[\mathbf{T}_1^2][\mathbf{0}_2][\mathbf{T}_3^2] \\ \Omega R\theta : \quad & [\Pi_{O7_3}] = -[\mathbf{T}_1^2][\mathbf{T}_2^2][\mathbf{0}_3] \end{aligned} \quad (5)$$

where the total effect is the sum of the above O-planes: $[\Pi_{O_p}] = [\Pi_{O3}] + [\Pi_{O7_1}] + [\Pi_{O7_2}] + [\Pi_{O7_3}]$.

2.2 The Fermionic Spectrum

Sector	Representation
aa	$U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets
$ab + ba$	$\mathcal{M}(\frac{N_a}{2}, \frac{\overline{N_b}}{2}) = I_{ab} = \prod_{i=1}^3 (n_a^i m_b^i - n_b^i m_a^i)$
$ab' + b'a$	$\mathcal{M}(\frac{N_a}{2}, \frac{N_b}{2}) = I_{ab'} = -\prod_{i=1}^3 (n_a^i m_b^i + n_b^i m_a^i)$
$aa' + a'a$	$\mathcal{M}(\text{Anti}_a) = \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO})$ $\mathcal{M}(\text{Sym}_a) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO})$

Table 1: Spectrum of bi-fundamental representations, where $I_{aa'} = -8 \prod_{i=1}^3 n_a^i m_a^i$, and $I_{aO} = 8(-m_a^1 m_a^2 m_a^3 + m_a^1 n_a^2 n_a^3 + n_a^1 m_a^2 n_a^3 + n_a^1 n_a^2 m_a^3)$.

Chiral matter arises from open strings with two ends attaching on different stacks. The multiplicity (\mathcal{M}) of the corresponding bi-fundamental representation is given by the ‘intersection’ number (as in Type IIA theory) between different stacks of branes. The initial $U(N_a)$ gauge group supported by a stack of N_a identical D6-branes is broken down by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to a subgroup $U(N_a/2)$. However a model may contain additional non-chiral (vector-like) multiplet pairs from $ab + ba$, $ab' + b'a$, and $aa' + a'a$ if the branes are parallel on at least one torus. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the intersection product, neglecting the null sector. For example, if $(n_a^1 m_b^1 - n_b^1 m_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] = \prod_{i=1}^3 (n_a^i m_b^i - n_b^i m_a^i)$,

$$\mathcal{M} \left[\left(\frac{N_a}{2}, \frac{\overline{N_b}}{2} \right) + \left(\frac{\overline{N_a}}{2}, \frac{N_b}{2} \right) \right] = \prod_{i=2}^3 (n_a^i m_b^i - n_b^i m_a^i) \quad (6)$$

The multiplicity of bi-fundamental as well as symmetric and antisymmetric representations are shown in Table 1.

2.3 Turning on Type IIB Fluxes

Turning on supergravity fluxes for closed string fields provides a possible way to stabilize the compactification moduli; however it also naturally breaks space-time supersymmetry in the bulk as well as contribute to the RR charges. Thus, specific solutions are needed to preserve supersymmetry.

The Type IIB non-trivial RR 3-form F_3 and NSNS 3-form H_3 fluxes compactified on Calabi-Yau threefold X_6 need to obey the Bianchi identities and be quantized [19]:

$$dF_3 = 0, \quad dH_3 = 0 \quad (7)$$

$$\frac{1}{(2\pi)^2\alpha'} \int_{X_6} F_3 \in \mathbf{Z}, \quad \frac{1}{(2\pi)^2\alpha'} \int_{X_6} H_3 \in \mathbf{Z} \quad (8)$$

When the two fluxes are turned on, they induce a covariant field $G_3 = F_3 - \tau H_3$ and contribute to the D3-brane RR charges

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2} \int_{X_6} H_3 \wedge F_3 = \frac{1}{(4\pi^2\alpha')^2} \frac{i}{2\text{Im}(\tau)} \int_{X_6} G_3 \wedge \bar{G}_3 \quad (9)$$

where $\tau = a + i/g_s$ being the Type IIB axion-dilaton coupling.

A complex cohomology basis can be utilized to describe the 3-form flux G_3 on $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$:

$$\begin{aligned} \omega_{B_0} &= dz^1 \wedge dz^2 \wedge dz^3, & \omega_{A_1} &= d\bar{z}^1 \wedge dz^2 \wedge dz^3, \\ \omega_{B_1} &= dz^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3, & \omega_{A_2} &= dz^1 \wedge d\bar{z}^2 \wedge dz^3, \\ \omega_{B_2} &= d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^3, & \omega_{A_3} &= dz^1 \wedge dz^2 \wedge d\bar{z}^3, \\ \omega_{B_3} &= d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3, & \omega_{A_0} &= d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3 \end{aligned} \quad (10)$$

where $dz^i = dx^i + U_i dy^i$, U_i are complex structure moduli. Here ω_{B_0} corresponds to the (3,0) of the flux, ω_{B_i} with $i=1, 2, 3$ correspond to (1,2) of the flux, ω_{A_i} with $i=1, 2, 3$ correspond to (2,1), and ω_{A_0} is (0,3) component of the flux. Then the untwisted 3-form G_3 takes the form:

$$\frac{1}{(2\pi)^2\alpha'} G_3 = \sum_{i=0}^3 (A^i \omega_{A_i} + B^i \omega_{B_i}) \quad (11)$$

Therefore the contribution of the fluxes to the RR tadpole condition N_{flux} can be calculated in terms of the basis defined above:

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2} \frac{i}{2\text{Im}(\tau)} \int_{X_6} G_3 \wedge \bar{G}_3 = \frac{4 \prod_{i=1}^3 \text{Im}(U^i)}{\text{Im}(\tau)} \sum_{j=0}^3 (|A^j|^2 - |B^j|^2) \quad (12)$$

The choice of fluxes may be positive (ISD-fluxes¹) or negative (IASD-fluxes). However, in order to satisfy the supergravity equation of motion, the BPS-like self-dual condition $*_6 G_3 = i G_3$ demands N_{flux} to be positive [20, 22, 23]. The quantization conditions of F_3 and H_3 fluxes require that N_{flux} be a multiple of 64.

¹Imaginary self dual fluxes, lead to zero or negative cosmological constant(to lowest order).

2.4 Supersymmetry Conditions

$D = 4$ $N = 1$ supersymmetric vacua from flux compactification require $1/4$ supercharges of the ten-dimensional Type I theory be preserved both in the open and closed string sectors [23]. The supersymmetry constraints in the open string sector are from the world-volume magnetic field and those in the closed string sector induced by the fluxes.

2.4.1 Supersymmetry Conditions in the Closed String Sector

In the closed string sector, to ensure that the RR and NSNS fluxes are supersymmetric, the primitivity condition $G_3 \wedge J = 0$ should be satisfied [20]. Here J is the general Kähler form of $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [25]:

$$J = J_1 dz^1 \wedge d\bar{z}^1 + J_2 dz^2 \wedge d\bar{z}^2 + J_3 dz^3 \wedge d\bar{z}^3 \quad (13)$$

We list a few solutions below. We also require that the turned on fluxes are as small as possible to avoid too large RR charge and satisfy the above requirements.

(2, 1)-Flux

(1) A specific supersymmetric solution for G_3 is (2, 1)-form given in [22] as

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = -4\omega_{A_2} - 4\omega_{A_3} \quad (14)$$

where the complex structure U^i and the dilaton coupling τ stabilize at $U^1 = U^2 = U^3 = \tau = i$. This solution gives the flux RR tadpole contribution:

$$N_{flux} = 128 \quad (15)$$

(2) Another specific supersymmetric solution for (2, 1)-form is given in [25] as

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \frac{8}{\sqrt{3}} e^{-\pi i/6} (\omega_{A_1} + \omega_{A_2} + \omega_{A_3}) \quad (16)$$

The fluxes stabilize the complex structure toroidal moduli at values $U^1 = U^2 = U^3 = \tau = e^{2\pi i/3}$. Thus, the flux contributes to the RR tadpole contribution an amount:

$$N_{flux} = 192 \quad (17)$$

Non-SUSY This solution has the smallest contribution to the D3 RR charge. Although it is not supersymmetric due to the existence of (0, 3) component, it is still worthy of study since we do not observe supersymmetry at low energies. The 3-form flux is

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = 2(\omega_{A_0} + \omega_{A_1} + \omega_{A_2} + \omega_{A_3}) \quad (18)$$

with $U^1 = U^2 = U^3 = \tau = i$. The flux induced RR charge is then

$$N_{flux} = 64 \quad (19)$$

2.4.2 Supersymmetry Conditions in the Open String Sector

In order to preserve $N = 1$ supersymmetry in the open string sector, a constraint must be placed upon the D-brane world-volume magnetic fields $F^i = n^i/m^i\chi^i$ associated with each two-torus \mathbf{T}_i^2 which can be expressed in terms of an ‘angle’ θ_i (as in the Type IIA picture) on each torus, as $\sum_i \theta_i = 0 \bmod 2\pi$ [25], where $\tan\theta_i = (F^i)^{-1} = \frac{m^i\chi^i}{n^i}$ and $\chi^i = R_1^i R_2^i$ the area of the \mathbf{T}_i^2 in α' units. Then we can write it in a form that is similar to the constraints in Type IIA picture as [11]

$$\begin{aligned} & -x_A m_a^1 m_a^2 m_a^3 + x_B m_a^1 n_a^2 n_a^3 + x_C n_a^1 m_a^2 n_a^3 + x_D n_a^1 n_a^2 m_a^3 = 0 \\ & -n_a^1 n_a^2 n_a^3 / x_A + n_a^1 m_a^2 m_a^3 / x_B + m_a^1 n_a^2 m_a^3 / x_C + m_a^1 m_a^2 n_a^3 / x_D < 0 \end{aligned} \quad (20)$$

where $x_A = \lambda$, $x_B = \lambda/\chi^2\chi^3$, $x_C = \lambda/\chi^1\chi^3$, $x_D = \lambda/\chi^1\chi^2$, and λ is a normalization constant used to keep the variables on an equal footing.

2.5 RR Tadpole Cancellation and K-theory Constraints

The RR charges of the magnetized D-brane associated homology classes and the contribution from the orientifold planes as well as the effect of the fluxes must be cancelled, namely we demand

$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi'_a] + \sum_p N_{O_p} Q_{O_p} [\Pi_{O_p}] + N_{flux} = 0 \quad (21)$$

where $[\Pi_{O_p}]$ are the sum of the orientifold planes listed in (5), and $N_{O_p} Q_{O_p} = -32$ in D_p -branes for S_p -type O-planes. N_{flux} is the amount of flux turned on, and is quantized in units of the elementary flux as discussed above [22, 23, 25]. Filler branes wrapping cycles along the O-planes can also be introduced here to reduce the difficulty of satisfying this condition. Thus the RR tadpole cancellation condition can be simplified as

$$\begin{aligned} & -N^{(O3)} - \sum_a N_a n_a^i n_a^2 n_a^3 - \frac{1}{2} N_{flux} = -16 \\ & -N^{(O7_1)} + \sum_a N_a n_a^1 m_a^2 m_a^3 = -16 \\ & -N^{(O7_2)} + \sum_a N_a m_a^1 n_a^2 m_a^3 = -16 \\ & -N^{(O7_3)} + \sum_a N_a m_a^1 m_a^2 n_a^3 = -16 \end{aligned} \quad (22)$$

In addition to the RR-tadpole condition the discrete D-brane RR charges classified by \mathbb{Z}_2 K-theory groups in the presence of orientifolds, which are invisible by the ordinary homology [26, 39, 40, 41], should be also taken into account [26, 40].

In Type I superstring theory there exist non-BPS D-branes carrying non-trivial K-theory \mathbb{Z}_2 charges. To avoid this anomaly it is required that in compact spaces these non-BPS branes must exist in an even number [40]. In Type IIB picture, these Type I non-BPS p -branes can be regarded as a pair of Dp -brane and its world-sheet parity image. For example, $\widehat{D7}|_I = (\overline{D7} + \overline{D7}/\Omega)|_{IIB}$. We need to consider the effects both from D3-

and D7-branes since they do not contribute to the standard RR charges. The K-theory conditions for a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold were derived in [26] and are given by

$$\begin{aligned} \sum_a N_a m_a^1 m_a^2 m_a^3 &= 0 \bmod 4, & \sum_a N_a m_a^1 n_a^2 n_a^3 &= 0 \bmod 4, \\ \sum_a N_a n_a^1 m_a^2 n_a^3 &= 0 \bmod 4, & \sum_a N_a n_a^1 n_a^2 m_a^3 &= 0 \bmod 4. \end{aligned} \quad (23)$$

Furthermore, D-brane states are classified by the K-theory group due to the presence of NSNS 3-form fluxes as well. This requires adding additional D-branes to preserve the homological charges and the possibility of instanton mediating D-branes and fluxes [25]. These properties do not affect the main constraints, and they are not presently well known and need further study.

2.6 The Green-Schwarz Mechanism for Flipped $SU(5)$ GUT Construction

Although the total non-Abelian anomaly cancels automatically when the RR-tadpole conditions are satisfied, additional mixed anomalies like the mixed gravitational anomalies which generate massive fields are not trivially zero [11, 42]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. The couplings of the four untwisted Ramond-Ramond forms B_2^i to the $U(1)$ field strength F_a are [6]

$$\begin{aligned} N_a m_a^1 n_a^2 n_a^3 \int_{M4} B_2^1 \wedge \text{tr} F_a, & \quad N_a n_a^1 m_a^2 n_a^3 \int_{M4} B_2^2 \wedge \text{tr} F_a \\ N_a n_a^1 n_a^2 m_a^3 \int_{M4} B_2^3 \wedge \text{tr} F_a, & \quad -N_a m_a^1 m_a^2 m_a^3 \int_{M4} B_2^4 \wedge \text{tr} F_a \end{aligned} \quad (24)$$

These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. In flipped $SU(5) \times U(1)_X$, the symmetry $U(1)_X$ must remain a gauge symmetry so that it may remix to help generate the standard model hypercharge after the breaking of $SU(5)$. Therefore, we must ensure that the gauge boson of the flipped $U(1)_X$ group does not receive such a mass. The $U(1)_X$ is a linear combination of the $U(1)$ s from each stack :

$$U(1)_X = \sum_a c_a U(1)_a \quad (25)$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand :

$$\begin{aligned} \sum_a c_a N_a m_a^1 n_a^2 n_a^3 &= 0, & \sum_a c_a N_a n_a^1 m_a^2 n_a^3 &= 0 \\ \sum_a c_a N_a n_a^1 n_a^2 m_a^3 &= 0, & \sum_a c_a N_a m_a^1 m_a^2 m_a^3 &= 0 \end{aligned} \quad (26)$$

The G-S mechanism will be considered only after the coefficients of $U(1)_X$ are determined.

3 Flipped $SU(5) \times U(1)_X$ Model Building

In the previous section we have outlined all the necessary machinery for constructing models as Type IIB flux vacua on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Our goal now is to realize a supersymmetric $SU(5) \times U(1)_X$ gauge theory with three generations and a complete GUT and electroweak Higgs sector in the four-dimensional spacetime. We also try to avoid as much extra matter as possible.

3.1 Basic Flipped $SU(5)$ Phenomenology

In a flipped $SU(5) \times U(1)_X$ [1, 31, 32] unified model, the electric charge generator Q is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where Y' is the $U(1)$ internal $SU(5)$ and \tilde{Y} is the external $U(1)_X$ factor. Essentially, this means that the photon is ‘shared’ between $SU(5)$ and $U(1)_X$. The Standard Model (SM) plus right handed neutrino states reside within the representations **5**, **10**, and **1** of $SU(5)$, which are collectively equivalent to a spinor **16** of $SO(10)$. The quark and lepton assignments are flipped by $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\mathbf{5}, -\frac{3}{2}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L ; \quad F_{\mathbf{10}, \frac{1}{2}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L d_L^c \nu_L^c \right); \quad l_{\mathbf{1}, \frac{5}{2}} = e_L^c \quad (27)$$

In particular this results in the **10** containing a neutral component with the quantum numbers of ν_L^c . We can spontaneously break the GUT symmetry by using a **10** and $\bar{\mathbf{10}}$ of superheavy Higgs where the neutral components provide a large vacuum expectation value, $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle$,

$$H_{\mathbf{10}, \frac{1}{2}} = \{Q_H, d_H^c, \nu_H^c\}; \quad \bar{H}_{\bar{\mathbf{10}}, -\frac{1}{2}} = \{Q_{\bar{H}}, d_{\bar{H}}^c, \nu_{\bar{H}}^c\}. \quad (28)$$

The electroweak spontaneous breaking is generated by the Higgs doublets H_2 and \bar{H}_2

$$h_{\mathbf{5}, -1} = \{H_2, H_3\}; \quad \bar{h}_{\bar{\mathbf{5}}, 1} = \{\bar{H}_2, \bar{H}_3\} \quad (29)$$

Flipped $SU(5)$ model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2)-triplet(H_3) splitting problem of the electroweak Higgs pentaplets h, \bar{h} through the trilinear coupling of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \rightarrow \langle \nu_H^c \rangle d_H^c H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields ϕ , $F_{\mathbf{10}} \cdot \bar{H}_{\bar{\mathbf{10}}} \cdot \phi \rightarrow \langle \nu_H^c \rangle \nu^c \phi$.

The generic superpotential W for a flipped $SU(5)$ model will be of the form :

$$\lambda_1 F F h + \lambda_2 F \bar{f} \bar{h} + \lambda_3 \bar{f} l^c h + \lambda_4 F \bar{H} \phi + \lambda_5 H H h + \lambda_6 \bar{H} \bar{H} \bar{h} + \dots \in W \quad (30)$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism [1].

4 Some Models with Fluxes

4.1 $N_{flux} = 192$

The most ideal situation is to preserve supersymmetry both in the closed string and open string sectors in the spirit of this flux construction. However we found that it is difficult to achieve. An example of this is shown in Table 2. Although this example is supersymmetric both in the open and closed string sectors, satisfies the conditions for cancellation of RR charges, and yields a three generation flipped $SU(5)$ model with a complete but extended Higgs sector, it does not satisfy the K-theory constraints.

stk	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$D7_2$
a	10	(1, 0)	(-1, -1)	(-2, 1)	2	-2	-12	24	1	-3	1	-3	0(1)	-2	0(1)	-2	2
b	2	(3, -1)	(-5, 1)	(4, -1)	332	148	-	-	7	15	7	15	12	16	12	16	12
c	2	(-2, 1)	(2, 1)	(-1, 0)	0	0	-	-	-	-	0(0)	0(16)	0(0)	0(9)	0(0)	0(9)	2
d	2	(-2, 1)	(2, 1)	(-1, 0)	0	0	-	-	-	-	-	-	0(0)	0(9)	0(0)	0(9)	2
e	2	(-1, 1)	(1, 1)	(-1, 0)	0	0	-	-	-	-	-	-	-	-	0(0)	0(4)	1
f	2	(-1, 1)	(1, 1)	(-1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	1
$O7_2$	6	(0, 1)	(1, 0)	(0, -1)	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 2: List of wrapping numbers and intersection numbers for three-fluxes $N_{flux} = 192$. The number in parenthesis indicates the multiplicity of non-chiral pairs. Here $x_A = 62$, $x_B = 1$, $x_C = 1$, and $x_D = 2$. It is obvious that the first K-theory constraint is not satisfied. The gauge symmetry is $U(5) \times U(1)^5 \times USp(6)$.

4.2 $N_{flux} = 128$

We present an example for $N_{flux} = 128$ with four stacks of magnetized D-branes as well as two filler branes presented in Table 3. Although this particular model does not contain flipped $SU(5)$ symmetry, it is a consistent solution of the RR tadpole conditions and the K-theory constraints, and is supersymmetric both in the open and closed string sectors. The gauge symmetry is

$$U(5) \times U(1) \times USp(4) \times USp(4) \quad (31)$$

stk	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	A	S	b	b'	$D3$	$D7_2$
a	10	(1, 0)	(-1, -1)	(-2, 1)	2	-2	-16	24	0(1)	2
b	2	(3, -2)	(-3, 1)	(4, -1)	374	202	-	-	-2	12
$O3$	4	(1, 0)	(1, 0)	(1, 0)	-	-	-	-	-	-
$O7_2$	4	(0, 1)	(1, 0)	(0, -1)	-	-	-	-	-	-

Table 3: $N_{flux} = 128$. The number stacks is only two plus two filler branes, though it has very few exotic particles, we have too few stacks to complete the cancellation of $U(1)_X$ mass. Here $x_A = 27$, $x_B = 1$, $x_C = 1$, and $x_D = 2$.

4.3 $N_{flux} = 1 \times 64$

stk	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	10	(1, 0)	(-1, -1)	(-2, 1)	2	-2	-8	12	-8	12	0(0)	0(8)	0(1)	4	0(1)	4
b	2	(1, -1)	(-3, 1)	(4, -1)	84	12	-	-	0(0)	96	8	12	4	0(6)	4	0(6)
c	2	(1, -1)	(-3, 1)	(4, -1)	84	12	-	-	-	-	8	12	4	0(6)	4	0(6)
d	2	(-1, 0)	(1, 1)	(-2, 1)	2	-2	-	-	-	-	-	-	0(1)	4	0(1)	4
e	2	(1, 1)	(1, 0)	(2, -1)	2	-2	-	-	-	-	-	-	-	-	0(0)	8
f	2	(1, 1)	(1, 0)	(2, -1)	2	-2	-	-	-	-	-	-	-	-	-	-

Table 4: List of intersection numbers for $N_{flux} = 64$ with gauge group $U(5) \times U(1)^5$. The number in parenthesis indicates the multiplicity of non-chiral pairs.

In this example, we use two sets of parallel D-branes and all conditions are satisfied. No filler brane is needed, and $x_A = 22$, $x_B = 1$, $x_C = 1$, and $x_D = 2$. The complete (n_a^i, m_a^i) and $SU(5) \times U(1)_X$ spectrum are listed in Table 4 and 5, and $U(1)_X$ is

$$U(1)_X = \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c - 5U(1)_d + 5U(1)_e - 5U(1)_f) \quad (32)$$

The four global $U(1)$ s from the Green-Schwarz mechanism are given respectively:

$$\begin{aligned}
U(1)_1 &= 24U(1)_b + 24U(1)_c + 4U(1)_e + 4U(1)_f \\
U(1)_2 &= 20U(1)_a + 8U(1)_b + 8U(1)_c + 4U(1)_d \\
U(1)_3 &= -10U(1)_a + 6U(1)_b + 6U(1)_c - 2U(1)_d - 2U(1)_e - 2U(1)_f \\
U(1)_4 &= -2U(1)_b - 2U(1)_c
\end{aligned} \quad (33)$$

From Table 5 we found that none of the global $U(1)$ s from the G-S anomaly cancellation mechanism provides Yukawa couplings required for generation of mass terms in superpotential (30). However, $U(1)_X$ admits these Yukawa couplings, and if we require the other anomaly-free and massless combination $U(1)_Y$ does as well, two conditions can be considered. The first one is to demand all the Yukawa couplings from the assigned intersections, and an example of the $U(1)_Y$ and the corresponding combinations of representations are listed as follows:

$$U(1)_Y^1 = 5U(1)_a - 25U(1)_b + 25U(1)_c - 25U(1)_d - 38U(1)_e + 38U(1)_f \quad (34)$$

$$\begin{aligned}
FFh &\rightarrow (\mathbf{10}, \mathbf{1})(\mathbf{10}, \mathbf{1})(\mathbf{5}_a, \mathbf{1}_d)^* \\
F\bar{f}\bar{h} &\rightarrow (\mathbf{10}, \mathbf{1})(\mathbf{\bar{5}}_a, \mathbf{1}_b)(\mathbf{\bar{5}}_a, \mathbf{\bar{1}}_d)^* \\
\bar{f}l^c h &\rightarrow (\mathbf{\bar{5}}_a, \mathbf{1}_b)(\mathbf{1}_c, \mathbf{\bar{1}}_d)(\mathbf{5}_a, \mathbf{1}_d)^* \\
F\bar{H}\phi &\rightarrow (\mathbf{10}, \mathbf{1})(\mathbf{\bar{10}}, \mathbf{1})(\mathbf{1}_b, \mathbf{1}_c) \\
HHh &\rightarrow (\mathbf{10}, \mathbf{1})(\mathbf{10}, \mathbf{1})(\mathbf{5}_a, \mathbf{1}_d)^* \\
\bar{H}\bar{H}\bar{h} &\rightarrow (\mathbf{\bar{10}}, \mathbf{1})(\mathbf{\bar{10}}, \mathbf{1})(\mathbf{\bar{5}}_a, \mathbf{\bar{1}}_d)^*
\end{aligned} \quad (35)$$

If we do not require the Higgs pentaplet \bar{h}' coupled with the chiral fermions in the term $F\bar{f}\bar{h}'$ to be the same as the Higgs pentaplet \bar{h} coupled to \bar{H} , then we expect a mixture state $\bar{h}_x = c\bar{h}' + s\bar{h}$ of these two different Higgs pentaplets in the Higgs sector, therefore

$$U(1)_Y^2 = U(1)_b - U(1)_c + U(1)_e - U(1)_f \quad (36)$$

$$\begin{aligned}
F\bar{f}\bar{h}' &\rightarrow (\mathbf{10}, \mathbf{1})(\bar{\mathbf{5}}_a, \mathbf{1}_b)(\bar{\mathbf{5}}_a, \mathbf{1}_c) \\
\bar{H}\bar{H}\bar{h} &\rightarrow (\bar{\mathbf{10}}, \mathbf{1})(\bar{\mathbf{10}}, \mathbf{1})(\bar{\mathbf{5}}_a, \bar{\mathbf{1}}_d)^*
\end{aligned} \tag{37}$$

We should also notice that the superfluous $\bar{\mathbf{5}}, \mathbf{5}$, and $\bar{\mathbf{10}}$ representations may be ostracized from the low energy spectrum through trilinear couplings of the generic form $\bar{\mathbf{5}} \cdot \mathbf{5} \cdot \mathbf{1}$ and $\bar{\mathbf{10}} \cdot \mathbf{10} \cdot \mathbf{1}$ satisfying the gauged $U(1)$ symmetries, where the singlets are assumed to acquire string scale *vevs*.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_Y^1$	$U(1)_Y^2$
$(\mathbf{10}, \mathbf{1})$	3	2	0	0	0	0	0	1	0	40	-20	0	10	0
$(\bar{\mathbf{5}}_a, \mathbf{1}_b)$	3	-1	1	0	0	0	0	-3	24	-12	16	-2	-30	1
$(\mathbf{1}_c, \bar{\mathbf{1}}_d)$	3	0	0	1	-1	0	0	5	24	4	8	-2	50	-1
$(\mathbf{10}, \mathbf{1})$	1	2	0	0	0	0	0	1	0	40	-20	0	10	0
$(\bar{\mathbf{10}}, \mathbf{1})$	1	-2	0	0	0	0	0	-1	0	-40	20	0	-10	0
$(\bar{\mathbf{5}}_a, \mathbf{1}_d)^*$	1	1	0	0	1	0	0	-2	0	24	-12	0	-20	0
$^1(\bar{\mathbf{5}}_a, \bar{\mathbf{1}}_d)^*/^2\bar{h}_x$	1	$^1-1$	10	10	$^1-1$	10	10	2	10	$^1-24$	112	10	120	$^2-1/0$
$(\mathbf{1}_b, \mathbf{1}_c)$	4	0	1	1	0	0	0	0	48	16	12	-4	0	0
$(\bar{\mathbf{15}}, \mathbf{1})$	2	-2	0	0	0	0	0	-1	0	-40	20	0	-10	0
$(\bar{\mathbf{10}}, \mathbf{1})$	1	-2	0	0	0	0	0	-1	0	-40	20	0	-10	0
$(\bar{\mathbf{5}}_a, \mathbf{1}_b)$	5	-1	1	0	0	0	0	-3	24	-12	16	-2	-30	1
$(\bar{\mathbf{5}}_a, \mathbf{1}_b)$	12	1	1	0	0	0	0	-2	24	28	-4	-2	-20	1
$(\bar{\mathbf{5}}_a, \mathbf{1}_c)$	8	-1	0	1	0	0	0	2	24	-12	16	-2	20	-1
$(\bar{\mathbf{5}}_a, \mathbf{1}_c)$	12	1	0	1	0	0	0	3	24	28	-4	-2	30	-1
$(\bar{\mathbf{5}}_a, \mathbf{1}_e)$	4	1	0	0	0	1	0	3	4	20	-12	0	-33	1
$(\bar{\mathbf{5}}_a, \mathbf{1}_f)$	4	1	0	0	0	0	1	-2	4	20	-12	0	43	-1
$(\mathbf{1}_b, \mathbf{1}_c)$	92	0	1	1	0	0	0	0	48	16	12	-4	0	0
$(\mathbf{1}_b, \bar{\mathbf{1}}_d)$	8	0	1	0	-1	0	0	0	24	4	8	-2	0	1
$(\mathbf{1}_b, \mathbf{1}_d)$	12	0	1	0	1	0	0	-5	24	12	4	-2	-50	1
$(\mathbf{1}_b, \bar{\mathbf{1}}_e)$	4	0	1	0	0	-1	0	-5	20	8	8	-2	13	0
$(\mathbf{1}_b, \bar{\mathbf{1}}_f)$	4	0	1	0	0	0	-1	0	8	8	-2	0	-63	2
$(\mathbf{1}_c, \bar{\mathbf{1}}_d)$	5	0	0	1	-1	0	0	5	24	4	8	-2	50	-1
$(\mathbf{1}_c, \mathbf{1}_d)$	12	0	0	1	1	0	0	0	24	12	4	-2	0	-1
$(\mathbf{1}_c, \bar{\mathbf{1}}_e)$	4	0	0	1	0	-1	0	0	20	8	8	-2	63	-2
$(\mathbf{1}_c, \bar{\mathbf{1}}_f)$	4	0	0	1	0	0	-1	5	20	8	8	-2	-13	0
$(\mathbf{1}_d, \mathbf{1}_e)$	4	0	0	0	1	1	0	0	4	4	-4	0	-63	1
$(\mathbf{1}_d, \mathbf{1}_f)$	4	0	0	0	1	0	1	-5	4	4	-4	0	13	-1
$(\mathbf{1}, \mathbf{1})$	12	0	2	0	0	0	0	-5	48	16	12	-4	-50	2
$(\mathbf{1}, \mathbf{1})$	12	0	0	2	0	0	0	5	48	16	12	-4	50	-2
$(\bar{\mathbf{1}}, \bar{\mathbf{1}})$	2	0	0	0	-2	0	0	5	0	-8	4	0	50	0
$(\bar{\mathbf{1}}, \bar{\mathbf{1}})$	2	0	0	0	0	-2	0	-5	-8	0	4	0	76	-2
$(\bar{\mathbf{1}}, \bar{\mathbf{1}})$	2	0	0	0	0	0	-2	5	-8	0	4	0	-76	2
$(\bar{\mathbf{5}}_a, \mathbf{1}_d)^*$	7	1	0	0	1	0	0	-2	0	24	-12	0	-20	0
$(\bar{\mathbf{5}}_a, \bar{\mathbf{1}}_d)^*$	7	-1	0	0	-1	0	0	2	0	-24	12	0	20	0
Additional non-chiral Matter														

Table 5: The spectrum of $U(5) \times U(1)^5$, or $SU(5) \times U(1)_X \times U(1)_Y$, with the four global $U(1)$ s from the Green-Schwarz mechanism. The \star/d representations indicate vector-like matter. We list the two cases for the $U(1)_Y$.

5 Conclusions

In this paper, we built flipped $SU(5)$ GUT models using D-brane constructions on a Type IIB $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with supergravity fluxes turned on. We considered both supersymmetric and non-supersymmetric fluxes in the closed string sector, and we claim that only the non-supersymmetric (soft-breaking) cases of flipped $SU(5)$ we have found are consistent with all the constraints of string theory including K-theory and supersymmetry in the open string sector.

The model that we have presented in Table 5 contains three-generations of chiral fermions and a complete GUT and electroweak Higgs sector. It also includes extra matter such as two copies of the symmetric representation of $SU(5)$ as well as many extra bi-fundamental and vector-like representations, which result from the large D9-brane co-prime numbers $(n_a^i, m_a^i)_{D9_a}$ needed for the required compensation of the induced three-form flux contributions to the D3 RR charge.

As mentioned above, the non-supersymmetric flux ($N_{flux} = 64$) in this particular flipped $SU(5)$ model breaks supersymmetry in the closed string sector. This leads to a mechanism of soft supersymmetry breaking at a mass scale $M_{soft} \sim \frac{M_{string}^2}{M_{Pl}}$ which implies an intermediate string scale or an inhomogeneous warp factor in the internal space to stabilize the electroweak scale [21, 22, 23]. With this non-supersymmetric flux present, soft supersymmetry breaking terms may be manifested in the effective action of open string fields. Detailed studies in soft-breaking mechanism and some trial investigations into the effective low energy scenario were studied in [22, 28]. Combined with a Yukawa coupling analysis [29], this may provide a clear picture of the low energy physics which we defer for future work.

The four global $U(1)$ symmetries from the G-S anomaly cancellation forbid all the Yukawa couplings necessary for the generation of quark and lepton masses, although if we ignore these global $U(1)$ factors and focus only on the $U(1)_X$ and $U(1)_Y$ symmetries, then we find that all of the required Yukawa couplings in (30) are present, as well as those needed for making the extra matter in the model obtain mass $\mathcal{O}(M_{string})$. We need to keep in mind that global $U(1)$ symmetries are valid to all orders in perturbation theory, and can be broken by non-perturbative instanton effects [43]. To solve this problem without these instanton effects, one possibility one may entertain is to use singlets, suitably charged, to trigger spontaneous breaking of global $U(1)$ s as well as of the local $U(1)_Y$ at the string scale, while leaving $U(1)_X$ intact. In the case of global $U(1)$ s one may hope that we will end up with invisible axion-like bosons. The interested reader may check from Table 5 that such singlets with appropriate charges do exist. Another possibility is that we may need a new D-brane configuration. It has been recently shown that the RR, NSNS and metric fluxes could contribute negative D6-brane charges in the Type IIA orientifold with flux compactifications, and thus relax the RR tadpole cancellation conditions [30], which is a good basis for future work as well as providing a solution to the problem of finding a compatible set of global $U(1)$ s on $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with the Yukawa couplings.

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